2. $y^3 y' + x^3 = 0$

```
ClearAll["Global`*"]

eqn = y[x]^3 + x^3 == 0;

sol = DSolve[eqn, y, x]
```

$\left\{\left\{y \to \text{Function}[\{x\}, -x]\right\},\right.$
$\left.\left\{y \to \text{Function}\left[\{x\}, (-1)^{1/3} x\right]\right\}, \left\{y \to \text{Function}\left[\{x\}, -(-1)^{2/3} x\right]\right\}\right\}$

```
eqn /. sol[[1]]
True

eqn /. sol[[2]]
True

eqn /. sol[[3]]
True
```

3. $y' = \sec^2 y$

```
ClearAll["Global`*"]

eqn = y'[x] == Sec[y[x]]^2;
eqn2 = y'[x] Cos[y[x]]^2 == 1;
```

$$\text{eqn3} = y'[x] == \frac{2}{\text{Cos}[2\, y[x]] + 1};$$

$$\text{eqn4} = y'[x] == \frac{4}{\left(e^{-i\, y[x]} + e^{i\, y[x]}\right)^2};$$

```
sol = DSolve[eqn3, y, x]
```

$\left\{\left\{y \to \text{Function}\left[\{x\}, \text{InverseFunction}\left[2\left(\frac{1}{4}\text{Sin}[2\, \#1] + \frac{\#1}{2}\right) \&\right][2 x + C[1]]\right]\right\}\right\}$

The WolframAlpha solution is shown as

$$c_1 + 2 x = 2\left(\frac{y[x]}{2} + \frac{1}{4}\text{Sin}[2\, y[x]]\right)$$

The above cell, which is somewhat easier to understand than the Mathematica solution, agrees with the text (the text's $c = \frac{c_1}{2}$).

```
Simplify[eqn /. sol]
{True}
```

4. y' sin 2πx $= \pi$y cos 2πx

```
ClearAll["Global`*"]
```

```
eqn = y'[x] Sin[2 π x] == π y[x] Cos[2 π x];
sol = DSolve[eqn, y, x]
```

$\left\{\left\{y \rightarrow \text{Function}\left[\{x\}, \text{C}[1] \sqrt{\text{Sin}[2 \pi x]}\right]\right\}\right\}$

```
eqn /. sol
```

{True}

5. y y' $+ 36$ x $= 0$

```
ClearAll["Global`*"]
```

```
eqn = y[x] y'[x] + 36 x == 0;
sol = DSolve[eqn, y, x]
```

$\left\{\left\{y \rightarrow \text{Function}\left[\{x\}, -\sqrt{2} \sqrt{-18 x^2 + \text{C}[1]}\right]\right\},\right.$
$\left.\left\{y \rightarrow \text{Function}\left[\{x\}, \sqrt{2} \sqrt{-18 x^2 + \text{C}[1]}\right]\right\}\right\}$

```
eqn /. sol[[1]]
```

True

```
eqn /. sol[[2]]
```

True

```
ex = (-√2 √(-18 x² + C[1]))²
```

$2 \left(-18 x^2 + \text{C}[1]\right)$

```
y² - ex == 0
```

$y^2 - 2 \left(-18 x^2 + \text{C}[1]\right) == 0$

The above cell is equal to the text answer, with text's c=2C[1].

6. y' $= e^{2x-1} y^2$

```
ClearAll["Global`*"]
```

```
eqn = y'[x] == e^(2 x-1) y[x]²;
sol = DSolve[eqn, y, x]
```

$\left\{\left\{y \rightarrow \text{Function}\left[\{x\}, -\dfrac{2 e}{e^{2 x} + 2 e \text{C}[1]}\right]\right\}\right\}$

```
eqn /. sol
```

```
{True}
```

---

7. $xy' = y + 2 x^3 \sin^2 \frac{y}{x}$ (Set y/x = u)

```
ClearAll["Global`*"]
```

```
eqn = x y'[x] == y[x] + 2 x^3 Sin[ y[x]/x ]^2;
```

```
sol = DSolve[eqn, y, x]
```

Solve:ifun:

Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information ≫

$$\{\{y \to \text{Function}[\{x\}, -x \, \text{ArcCot}[x^2 - 2 \, C[1]]]\}\}$$

```
Simplify[eqn /. sol]
```

```
{True}
```

The Mathematica solution checks. However, assuming C[1]=0 for the purpose of checking, I find that

```
PossibleZeroQ[x ArcTan[x^2] + x ArcCot[x^2]]
```

```
False
```

the Mathematica answer does not seem to agree with that of the text.

8. $y' = (y + 4x)^2$ (Set $y + 4x = v$)

```
ClearAll["Global`*"]
```

```
eqn = y'[x] == (y[x] + 4 x)^2;
sol = DSolve[eqn, y, x]
```

$$\{\{y \to \text{Function}[\{x\}, -2 \, \mathbb{i} - 4 \, x + \frac{1}{-\frac{\mathbb{i}}{4} + e^{4 \, \mathbb{i} \, x} \, C[1]}]\}\}$$

```
Simplify[eqn /. sol]
```

```
{True}
```

---

9. $xy' = y^2 + y$ (Set $y/x = u$)

```
ClearAll["Global`*"]
```

```
eqn = x y'[x] == y[x]^2 + y[x];
sol = DSolve[eqn, y, x]
```

$$\{\{y \to \text{Function}[\{x\}, -\frac{e^{C[1]} \, x}{-1 + e^{C[1]} \, x}]\}\}$$

```
Simplify[eqn /. sol]
```

```
{True}
```

$$- \frac{e^{C[1]} x}{-1 + e^{C[1]} x} \;/.\; C[1] \to 0$$

$$- \frac{x}{-1 + x}$$

The text answer is $\frac{x}{c-x}$, which is slightly different from the Mathematica answer in yellow. However, the Mathematica solution does check out as valid.

10. $xy' = x + y$ (Set $y/x = u$)

```
ClearAll["Global`*"]
```

```
eqn = xy'[x] == x + y[x];
sol = DSolve[eqn, y, x]
```

```
{{y → Function[{x}, -x + xy′[x]]}}
```

```
eqn /. sol
```

```
{True}
```

11-17 Initial Value Problems (IVPs). Solve the IVP. Show the steps of derivation, beginning with the general solution.

11. $xy' + y = 0,\ y(4) = 6$

```
ClearAll["Global`*"]
```

```
eqn = x y'[x] + y[x] == 0;
sol = DSolve[{eqn, y[4] == 6}, y, x]
```

$$\left\{\left\{y \to \text{Function}\left[\{x\}, \frac{24}{x}\right]\right\}\right\}$$

```
eqn /. sol
```

```
{True}
```

The answer in green above matches the answer in the text.

12. $y' = 1 + 4y^2,\ y(1) = 0$

```
ClearAll["Global`*"]
```

```
eqn = y'[x] == 1 + 4 y[x]²;
sol = DSolve[{eqn, y[1] == 0}, y, x]
```

Solve:ifun:

Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information. ≫

$$\left\{\left\{y \rightarrow \text{Function}\left[\{x\}, \frac{1}{2} \text{Tan}[2 (-1 + x)]\right]\right\}\right\}$$

```
Simplify[eqn /. sol]
```

```
{True}
```

13. $y' \cosh^2 x = \sin^2 y, \ y(0) = \frac{1}{2}\pi$

```
ClearAll["Global`*"]
```

```
eqn = y'[x] Cosh[x]² == Sin[y[x]]²;
sol = DSolve[{eqn, y[0] == 1/2 π}, y, x]
```

Solve:ifun:

Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information. ≫

```
{{y → Function[{x}, -ArcCot[Tanh[x]]]}}
```

The answer in the green cell above matches that of the text.

```
Simplify[eqn /. sol]
```

```
{True}
```

14. $dr/dt = -2\,tr, \ r(0) = r_0$

```
ClearAll["Global`*"]
```

```
eqn = r'[t] == -2 t r[t];
sol = DSolve[{eqn, r[0] == r0}, r[t], t]
```

$$\left\{\left\{r[t] \rightarrow e^{-t^2} r0\right\}\right\}$$

```
Simplify[eqn] /. Simplify[sol]
```

$$\left\{r'[t] == -2\,e^{-t^2}\,r0\,t\right\}$$

While Mathematica will not declare this equality to be true, it works when the substitutions are made.

15. $y' = -4\,xy, \ y(2) = 3$

```
ClearAll["Global`*"]
```

```
eqn = y'[x] == -4 x y[x];
sol = DSolve[{eqn, y[2] == 3}, y, x]
```

$$\{\{y \to \text{Function}[\{x\}, 3\, e^{8-2\, x^2}]\}\}$$

```
eqn /. sol
{True}
```

Though the Mathematica solution does check out as valid (yellow), it does not agree with the text answer, which is $y^2 + 4 x^2 = c = 25$.

16. $y' = (x + y - 2)^2$, $y(0) = 2$, (Set v = x + y - 2)

```
ClearAll["Global`*"]
```

```
eqn = y'[x] == (x + y[x] - 2)^2;
sol = DSolve[{eqn, y[0] == 2}, y, x]
```
$$\{\{y \to \text{Function}[\{x\}, -(((-2-i)-(2-i)\, e^{2\, i\, x} + x + e^{2\, i\, x}\, x) / (1 + e^{2\, i\, x}))]\}\}$$

```
Simplify[eqn /. sol]
{True}
```

17. $xy' = y + 3 x^4 \cos^2\left(\frac{y}{x}\right)$, $y(1) = 0$, Set $\frac{y}{x} = u$

```
ClearAll["Global`*"]
```

```
eqn = x y'[x] == y[x] + 3 x^4 Cos[ y[x]/x ]^2;
sol = DSolve[{eqn, y[1] == 0}, y, x]
```

Solve:ifun:
 Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information ≫

$$\{\{y \to \text{Function}[\{x\}, -x\, \text{ArcTan}[1 - x^3]]\}\}$$

```
Simplify[eqn /. sol]
{True}
```

Though the Mathematica solution does check out as valid (yellow), it does not agree with the text answer, which is $y = x\, \text{ArcTan}[x^3 - 1]$

19 - 36 Modeling, Applications

19. Exponential growth. If the growth rate of the number of bacteria at any time t is proportional to the number present at t and doubles in 1 week, how many bacteria can be expected after 2 weeks? After 4 weeks?

21. Radiocarbon dating. What should be the $^{14}_{6}C$ content (in percent of y_0) of a fos-

silized tree that is claimed to be 3000 years old? (See example 4.)

23. Boyle-Mariotte's law for ideal gases. Experiments show for a gas at low pressure p (and constant temperature) the rate of change of the volume V(p) equals $\frac{-V}{p}$. Solve the model.
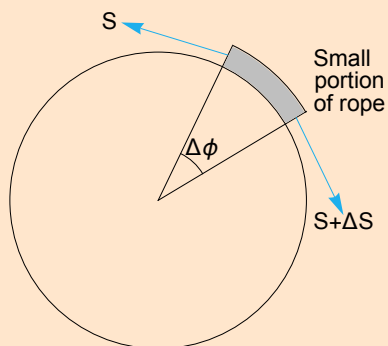
25. Newton's law of cooling. A thermometer, reading 5°C, is brought into a room whose temperature is 22°C. One minute later the thermometer reading is 12°C. How long does it take until the reading is practically 22°C, say, 21.9°C?

27. Dryer. If a wet sheet in a dryer loses its moisture at a rate proportional to its moisture content, and if it loses half of its moisture during the first 10 min of drying, when will it be practically dry, say, when will it have lost 99% of its moisture? First guess, then calculate.

29. Alibi? Jack, arrested when leaving a bar, claims that he has been inside for at least half an hour (which would provide him with an alibi). The police check the water temperature of his car (parked near the entrance of the bar) at the instant of arrest and again 30 min later, obtaining the values 190°F and 110°F, respectively. Do these results give Jack an alibi? (Solve by inspection.)

31. Solution curves of y' = g(y/x). Show that any (nonvertical) straight line through the origin of the xy-plane intersects all these curves of a given ODE at the same angle.

33. Rope. To tie a boat in a harbor, how many times must a rope be wound around a bollard (a vertical rough cylinder post fixed on the ground) so that a man holding one end of the rope can resist a force exerted by the boat 1000 times greater than the man can exert? First guess. Experiments show that the change Δs of the force S in a small portion of the rope is proportional to S and to the small angle Δϕ in the figure below. Take the proportionality constant 0.15. The result should surprise you!